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Engineering: On the Compaction of Particulate Materials, Like Granola, Etc.

In this column, I will return to a discussion that I began a long time ago. In my column published in June 2002 ("Comparing Rolling Granola Bars with Rolling Dough," *Cereal Foods World* 47:242-243), I presented a preliminary discussion of the similarities and differences between rolling granola bars and rolling dough. Since then, I have had a number of questions about this and have had some opportunity to look a little more closely at the physics, although I must admit that I still haven't completely solved all the equations that describe the process. A typical process is illustrated in Figure 1.

The sticky, particulate mixture containing grains and other particles that have been mixed with a binder, usually based on sugar syrups, is fed into a hopper above the slab-forming rolls. These rolls begin the initial compaction of the material. The thick "sheet" that is formed is fed to one or more combinations of a roller and belt, which complete the compaction of the sheet, compressing the material to the desired final density and thickness.

The question that faces us is, "What kind of forces does the material experience as it is compressed by the process?" These forces result in the compaction and shearing of the material but also may result in the breakage of friable components, such as rice crisps, which would result in an altered texture or the smearing of softer components, such as chocolate chips.

As stated in my previous column, this is not a problem of fluid flow, but rather a problem related to the physics of moving and transferring a particulate mixture. There is no simple viscosity model that can be applied to this kind of problem. Instead, we must use principles related to the actions of friction and pressure on the motion of solids.

In both slab-forming roll and belt/roll combinations, material moves as a result of the transfer of frictional force between the rolls and/or belts to a "slab" of material between the rolls or the roll and belt. In the simplest models, the forces are described by a "free body" diagram. Figure 2 shows free body diagrams for a slice of material anywhere in a two-roll or roll/belt system.

There are two key differences between the physics of these systems and systems undergoing viscous flow, such as the sheeting of doughs. 1) In viscous flow, we assume that the material sticks to the rolls, i.e., we assume that the velocity of material is equal to the velocity of the rolls at the point of contact. For particulate flow, the material can slip on the surface of the rolls. This is a significant complication, because depending on the physical conditions and the particular location between the rolls the material can move faster or slower than the surface of the rolls or belts. 2) In a viscous flow system, the velocity of the material within the slab is a function of the cross-sectional location, while in particulate flow the velocity within the slab is the same everywhere. This means that for the viscous flow case the shear on the product is a function of cross-sectional location but that for particulate flow shear may not be a function of cross-sectional location.

The first challenge is the estimation of shear stress (τ_{roll}) transferred from the roll surface to the slab of material. It is this shear that ultimately defines the conveyance efficiency of the system. To estimate shear stress one must deal with an equation that describes the angle of wall friction, which is another way of describing the coefficient of friction between the material and the roll and/or belt surface. The simplest equation that has been proposed to describe this shear stress is

$$\tau_{shear} = \tau_{adhesion} + \mu_f \sigma_{normal}$$

where τ_{shear} is the shear stress on the surface
$\tau_{adhesion}$ is the adhesion
μ_f is the coefficient of friction
σ_{normal} is the normal force on the surface

The leading term is adhesion, and it probably only comes into play for cohesive/sticky materials such as we are dealing with

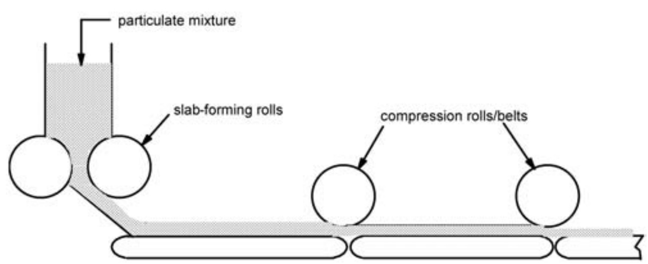

Fig. 1. The front end of a granola bar line.

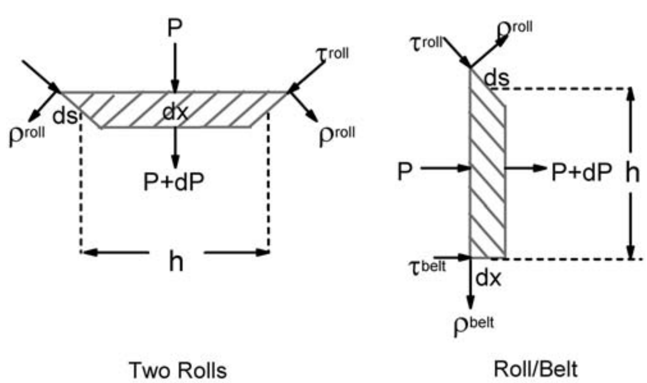

Fig. 2. Free body diagrams for two-roll and roll/belt systems.

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here. If adhesion is zero, the relationship between pressure and shear stress reduces to a simple linear relationship for friction, which is identical to the equation for Coulomb friction (i.e., frictional force = coefficient of friction × normal force). Without the adhesion term, if there is no pressure on the rolls no shear forces are transmitted between the rolls and/or belts and the slab of material. As a consequence, the material won't be conveyed. With adhesion, there is some conveyance force at the entrance of the system, but it might be too low to convey the desired quantity of material through the rolls.

Let's consider where the pressure comes from. For slab-forming rolls, this is obvious. The pressure at the first point of contact comes from the "head" of the material above the first point of contact. We can increase the pressure on the rolls by increasing the head (i.e., the amount of material [height] in the hopper above the rolls). This is why if one starts the rolls before the hopper is sufficiently filled, either no flow or a "broken" slab will emerge from the slab-forming rolls. Thus, controlling the height of the material in the hopper is critical for controlling the output flow from the slab-forming rolls. Poor control of the level of material in the hopper results in formation of a slab that has a variable degree of compaction (density).

The application of head is not as simple for solid/particulate flow as it is for liquid/viscous flow, however. For liquid flow, the head and, hence the pressure on the rolls at the point of first contact, increases linearly with liquid depth in the hopper. For solid flow, the head increases and reaches a maximum. Because adhesion also relates to the flow of material in the hopper, restricting flow through the hopper, the pressure maximum could be very small. If adhesion between the material and walls of the hopper is very high (the material is very sticky), then it is possible that the material would never be successfully conveyed through the rolls. Under these conditions, the rolls would have to be "precompacted" by force-feeding the rolls with something like a screw. I have never seen this used in the food industry but have heard of it being used in other industries.

Note, anything that increases the coefficient of friction between the rolls and material increases the shear stress being transferred, which in turn improves the conveyance capacity at the inlet of the system. This is one reason why one might consider using corrugated rolls in the slab-forming section of the system. Like the "rifling/grooving" of the feed zone in an extruder, corrugating the rolls increases the coefficient of friction between the material and rolls, improving the conveyance efficiency of the system.

The final roll/belt combinations that compress the slab to the desired final density and thickness present a different physical problem than the slab-forming rolls. There is no obvious pressure at the entry point to the roll/belt combination, and if there is no pressure and no adhesion between the rolls and/or belt, the rolls and/or belt would apparently not convey the material! Obviously, since these systems work, there must be pressure and/or adhesion at this point. Pressure could be created here by running the crude slab at a higher velocity than the roller. This would result in the creation of a small amount of pressure by converting some of the kinetic energy of the slab into pressure energy when it collides with the roll. This would also, of course, result in a "backup" and precompression of material behind the roll. I've not analyzed this situation, but I suspect that it might only be of limited value and might create additional problems. There is, however, another source of pressure at the point of first contact between the roll/belt combination. There is a small amount of head pressure on the belt as a result of the height of the material on the belt. The amount of pressure is very small, but for small compressions so is the pressure developed by the roll/belt combination, and it appears to play some role in the process. My guess is that the sticky nature of the material, expressed in the adhesion, is a critical factor in the conveyance of the material through the roll/belt combinations. One feature that suggests this is the fact that the belt and rolls are usually smooth, so one would expect that the coefficient of friction between the roll/belt and the material would be low. Since the coefficient of friction and the pressure developed are small, the shear result from simple friction would be small, and conveyance would prove very difficult without the added effect of adhesion.

The problem I have with further analysis of this system is the fact that there is simply no information available about the constants in the "angle of wall friction" equation given above. Without this information, no quantitative analysis is possible. I have sent notes to a number of people around the world, but no one has been able to lead me to this kind of information. Nonetheless, we can obtain a qualitative idea of how this equipment works by using "guesses" for some of the unknown constants and looking at relative effects. I have actually solved the flow equations for some situations, and the results are interesting. I will discuss this in one or more future columns.

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