Engineering: Residence Time Distribution III

In my two previous columns (Cereal Foods World, July-August 2008 and November-December 2008), I began a discussion of the concept of a residence time distribution, that is the distribution of times “particles” experience within a reaction vessel, for example an extruder, preconditioner, or any cooking/sterilization process. This column will continue my explanations with a description of how residence time distributions are obtained and analyzed.

As previously mentioned, residence time distributions are normally measured by introducing a “tracer” into the processing operation of interest. Although easily stated, the actual procedure is not as straightforward as it sounds. The tracer is normally introduced as an “instantaneous” pulse of material, but sometimes introduced as a step change in concentration in the feed, or even a sinusoidal variation in concentration in the feed. The most common is the pulse method which I will talk about here.

The first problem that arises is the choice of the tracer. Many things have been used such as colorants, salts, proteins, suspended particles, and radioactive materials. A suitable tracer should have the following characteristics:

1) Concentration of the tracer in the process output should be easily and accurately measured, preferably at infinitesimal levels.
2) The tracer should not alter the rheology of the system. Altering the rheology, say in a process like an extruder, will alter the flow patterns (velocity fields) within the process. This results in alteration (distortion) of the residence time distribution.
3) The tracer should undergo any reactions during passage through the process.
4) The tracer should be completely miscible in the material being processed. If the material is not completely miscible it can flow in a different path than the bulk of material, resulting in a distortion of the residence time distribution.

In food systems, one commonly uses intense colorants, which can be introduced and readily detected at very low levels. The ability to introduce the colorants at very low levels is desirable, because they will have little or no effect on the flow through the system. However, a common error appears in the literature when using colorants. Colorants exhibit a non-linear relationship between measured color, say with a colorimeter, and actual concentration of the tracer. So, color measurements must be converted to concentrations before analyzing the residence time distributions.

Assuming a suitable tracer is identified, the experimental process proceeds as follows:

1) The process is brought to a steady state operating condition.
2) The tracer is introduced as a pulse at a time set to zero. The introduction of a pulse may be done in various ways, such as the actual inject of a liquid into a feed stream, or dropping a small amount of concentrated pre-colored raw material into the feed of the machine. In extruders, a highly colored dough “ball” of feed material is sometimes dropped directly into the feed zone of the extruder where the dry feed is being introduced.
3) Samples of product exiting the process are frequently collected (the more often the better!) and analyzed for concentration of the tracer. The collection continues until there is no tracer detected at the exit.

The concentration data is now mathematically analyzed. To illustrate the analysis, I will consider the following data (Table I). (Only a limited amount of samples have been considered in order to simplify the illustration.)

The results must now be mathematically analyzed. First, calculate the total quantity of tracer introduced. This is done by integrating the concentration versus time.

\[
c_{\text{total}} = \int_0^{t_{\text{end}}} c \, dt \approx \sum_{n=0}^{n} c_{\Delta t}
\]

Now we scale the concentration data by dividing by \(c_{\text{total}}\). This is called the unit pulse response (Table II), generally called \(E(t)\). \(E(t)\) versus time is shown in Figure 1. In my previous column, this is what I called an un-normalized result. The area under the curve in Figure 1 is unity, so the area under the curve between any time \(t_1\) and \(t_2\), represents the probability (percentage) of material that will reside in the process between \(t_1\) and \(t_2\).

Now, we take the first moment of \(E(t)\) with time, which is the mean residence time, \(\tau\), by evaluating,
\[ \tau = \int_0^{t_{\text{end}}} t \cdot E(t) \, dt = \sum_{n = 0}^{n} (tE(t) \Delta t) \]

\[ \tau = 3.4 \]

So, on average, a “particle” of fluid stays in process for 3.4 min. A common error that is made is to simply look at the time when peak concentration is observed and call this the residence time. This would yield \( \tau = 3.0 \), which is incorrect.

The time data is now scaled by dividing the real time by the mean residence time, \( \tau \), to obtain \( \Theta \), the number of residences times that have passed. The unit pulse response \( E(t) \) is now scaled by multiplying by the mean residence time, \( \tau \), to obtain the dimensionless age distribution \( E(\Theta) \) (Table III).

A plot of this data is presented in Figure 2. The area under the curve is equal to unity. This is a true probability function, with a mean of \( \Theta = t/\tau = 1.0 \). The area under the curve between any number of residence times, \( \Theta_1 \) and \( \Theta_2 \), represents the probability (percentage) of material that will reside in the process between \( \Theta_1 \) and \( \Theta_2 \).

To describe a probability (age) distribution, one normally gives its variance. The variance with respect to the mean residence time, \( \tau(\Theta = 1) \), is given by

\[ \sigma^2 = \int_0^{t_{\text{end}}} \frac{1}{\tau} (\Theta - 1)^2 E(\Theta) \, d\Theta = \sum_{n = 0}^{n} \left[ (\Theta - 1)^2 E(\Theta) \Delta \Theta \right] \]

\[ \sigma^2 = 0.13 \]

The “quality” of this residence time distribution is now defined by this variance. A variance of zero represents a system which is perfectly plug flow (every particle stays in for 1 residence time). A variance of unity represents a completely mixed system (the time a particle stays in the system is completely random and is described by an exponential distribution).

Table II. Time (min)  | \( E(t) = \) concentration/\( c_{\text{total}} \)
---|---
0 | 0
1 | 0
2 | 0.268
3 | 0.317
4 | 0.232
5 | 0.122
6 | 0.049
7 | 0.012
8 | 0

Table III. \( t/\tau = \Theta \)  | \( E(\Theta) = \tau E(t) \)
---|---
0 | 0
0.294 | 0
0.588 | 0.913
0.882 | 1.079
1.176 | 0.415
1.470 | 0.166
1.763 | 0.098
2.057 | 0.024
2.351 | 0

Fig. 1. Non-normalized pulse response.

Fig. 2. Normalized pulse response (age distribution function).

In my next column, I will complete the discussion of residence time distributions by discussing how the residence time distributions interact with the kinetics on chemical and physical transformations occurring within the process.

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