DIFFUSION OF WATER IN KERNELS OF CORN AND SORGHUM[1]

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ABSTRACT

Diffusion rates of water in three varieties of corn, K-4 hybrid popcorn, Gold Rash sweet corn, and K-1859 hybrid corn, and two varieties of sorghum, white kafir and Atlas sorgo, were measured under temperatures ranging from 0° to 100°C. The approximate diffusion equation derived for particles with arbitrary shape was found valid at the initial period (up to approximately 2 hr.) of steeping grains in water at all the temperature levels. The diffusivities were evaluated from the data by use of the equation. The diffusivities were well correlated according to the Arrhenius law.

Corn and sorghum are two major sources of starch for industry. Generally, the starch is manufactured by a wet process. In the wet process, grains are softened by steeping in water (9,10). The diffusion of water into grains of corn and sorghum during steeping has been discussed in several papers (1,6,8). The heterogeneity of composition and irregularity in shape of the grains, however, make the diffusion problem a complicated one, and thus a quantitative and mathematical treatment of the diffusion rate of water into these grains is lacking.

The purpose of this work is to investigate the possibility of correlating such diffusion data quantitatively with the experimental variables.

Theoretical

The diffusion coefficient is defined by Fick's second law as:

$$D\left(\frac{\partial^2 c}{\partial x^2}+\frac{\partial^2 c}{\partial y^2}+\frac{\partial^2 c}{\partial z^2}\right)=\frac{\partial c}{\partial t} \qquad (1)$$

where D is the diffusion coefficient; c is the concentration of diffusing substance at a point in solid; x, y, and z are Cartesian coordinates of the point under consideration; and t is the diffusion time. It has been demonstrated that equation 1 can be integrated approximately for a particle of arbitrary shape (2,3,7):

$$\overline{C}=\frac{\overline{c}-c_s}{c_o-c_s}=1-\frac{2}{\sqrt{\pi}}X \qquad (2)$$

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in which $\bar{c}$ is the average concentration, c_0 the initial concentration and c_s the concentration at bounding surface, and $X = S/V \sqrt{Dt}$ in which S is the surface area and V is the volume. If the concentration term is represented by the moisture content on a weight basis:

$$\bar{C} = \bar{M} = \frac{\bar{m} - m_s}{m_0 - m_s} \quad (3)$$

in which $\bar{m}$ is the average moisture content at the given time, m_s is the effective surface moisture content, and m_0 is the initial moisture content (moisture contents expressed in g./g.). Equation 2, then, can be written as follows:

$$\bar{M} = 1 - \frac{2}{\sqrt{\pi}} X \quad (4)$$

or, in terms of experimental variables:

$$\bar{m} - m_0 = K\sqrt{t}$$

where

$$K = \frac{2}{\sqrt{\pi}}(m_s - m_0) \frac{S}{V} \sqrt{D} \quad (5)$$

The formulation outlined above is based on Becker's work and it has been shown that the equation can correlate the diffusion rate of water in wheat kernels under widely varied conditions (2,3,7). However, it has yet to be tested for other cereal grains.

Material and Method

The material used in this study was K-4 hybrid popcorn, K-1859 hybrid dent corn, Gold Rash sweet corn, white kafir sorghum, and Atlas sorgo sorghum. The characteristics of the material are listed in Table I.

The experiments for measuring the weight increase of the samples during diffusion were carried out over a temperature range of 0° to 100°C., and the periods of absorption time ranged from several minutes to 6 hr.

The details of the experimental procedure were reported previously (5,7).

Correlation and Discussion of Results

In Figs. 1 and 2, the weights of K-4 hybrid popcorn and white kafir grain sorghum are plotted as functions of the absorption time as illustrated. The results show that at the beginning the rates of weight increase for both samples were quite rapid, but they decreased gradually as the absorption time increased. Similar results were obtained for other samples.

TABLE I

CHARACTERISTICS OF MATERIAL USED

	K-4 HYBRID POPCORN	GOLD RASH SWEET CORN	K-1859 HYBRID DENT CORN	WHITE KAFIR GRAIN SORGHUM	ATLAS SORGO SORGHUM
	%	%	%	%	%
Protein	10.96	10.88	9.44	10.69	8.88
Ether extract	3.69	8.18	4.13	3.24	3.55
Crude fiber	3.25	1.99	1.99	1.87	1.76
Moisture	9.78	10.10	10.96	10.74	9.73
Ash	1.45	1.83	1.40	1.72	1.51
N-free extract	72.14	67.02	72.08	71.74	74.57
Carbohydrates	74.39	69.01	74.07	73.61	76.33
Average initial surface area per grain, $cm.^2$	1.0655	2.334	2.2745	0.3363	0.3193
Average initial volume per grain, $cm.^3$	0.1038	0.1874	0.2634	0.0150	0.0141
Density, $g./cm.^3$	1.333	1.3228	1.250	1.2755	1.3360

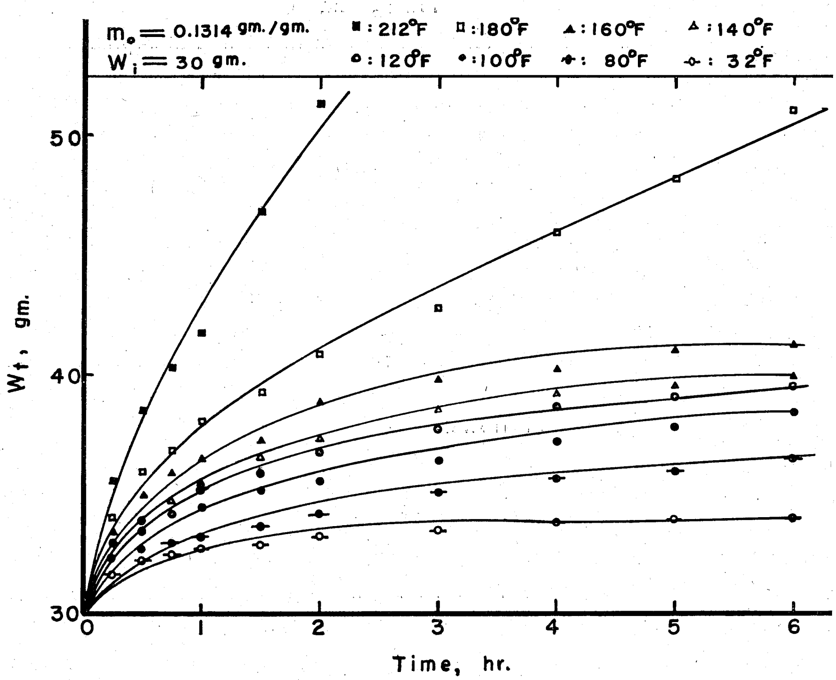

Fig. 1. Weight of popcorn as a function of absorption time.

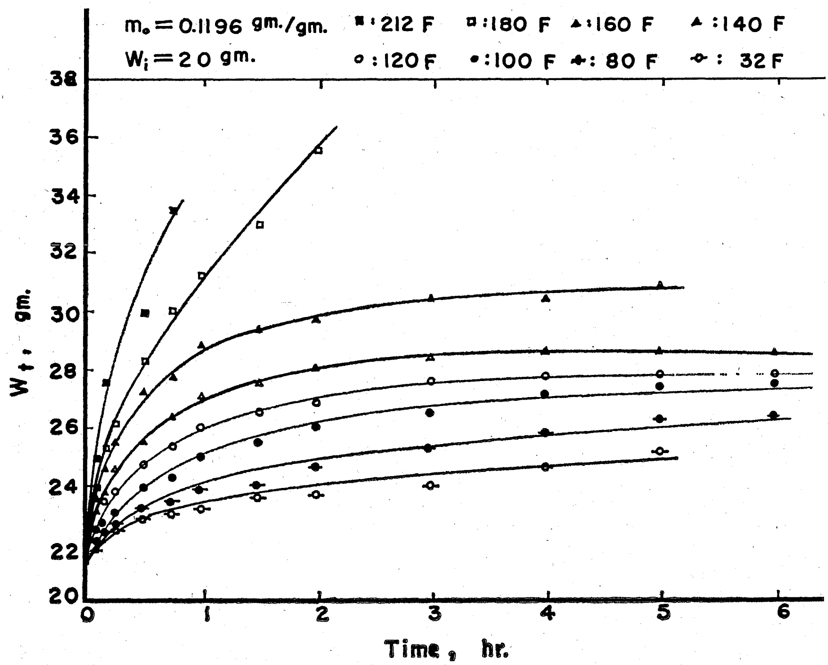

Fig. 2. Weight of white kafir (grain sorghum) as a function of absorption time.

Initial Moisture Weight Gain, m_i. In testing the fitness of the experimental data to the derived diffusion equation, the quantity of rapid initial weight gain, m_i, which has been known to be due to the phenomenon of capillary action, should be considered (3,7).

The quantity, m_i, was measured for each of the five different samples at temperature levels ranging from 0° to 100°C., assuming 5 sec. as the time needed for these samples to complete their capillary action. For corn, this m_i was nearly constant, but for sorghum m_i increased slightly with the temperature. The presence of a porous starch-containing layer not found in corn may explain the difference. The measured values of m_i were as follows:

0.020 for popcorn
0.016 for dent corn
0.029 for sweet corn
0.066 at 0°C. to 0.083 at 100°C. for white kafir sorghum
0.011 at 0°C. to 0.030 at 100°C. for Atlas sorgo sorghum

Effective Surface Moisture Content, m_s. The effective surface moisture content, m_s, was evaluated from the following considerations. In equation 5 the quantities (S/V) $\sqrt{Dt}$, m_s, and m_0 are all constant. Equation 5 can, therefore, be expressed as:

$$\overline{m} - m_o = K_A (m_s - m_o) \tag{6}$$

where

$$-K_A = \frac{2}{\sqrt{\pi}} \left(\frac{S}{V}\right) \sqrt{Dt} \tag{7}$$

Equations 6 and 7 show that, in case the diffusion time is kept constant, the quantity $(\overline{m} - m_0)$ should be a linear function of m_0 with a slope of $-K_A$ and $m_0 = m_s$ when $(\overline{m} - m_0)$ equals zero. On the basis of this relationship, the value of m_s could be experimentally evaluated.

A number of samples which had different initial moisture contents, m_0, were steeped in water for 15 min. to measure their weight gain. The experimental results are plotted in Fig. 3 for popcorn, and Fig. 4

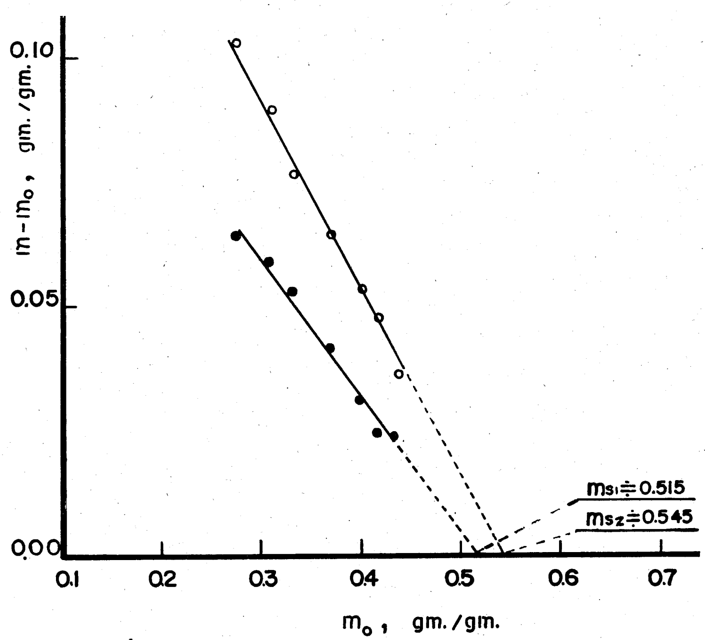

Fig. 3. Extrapolation of $m - m_0$ as function of m_0 at different temperature to obtain the effective surface moisture content m_s for the K-4 hybrid popcorn. Time, 15 min. Open circles, 160°F.; solid circles, 100°F.

for white kafir sorghum. These figures indicate that the linear relationship was obeyed by plotting $(\overline{m} - m_0)$ as a function of m_0. The values of m_s were evaluated by extrapolating the straight line to intercept with the abscissa, where $(\overline{m} - m_0)$ was equal to zero.

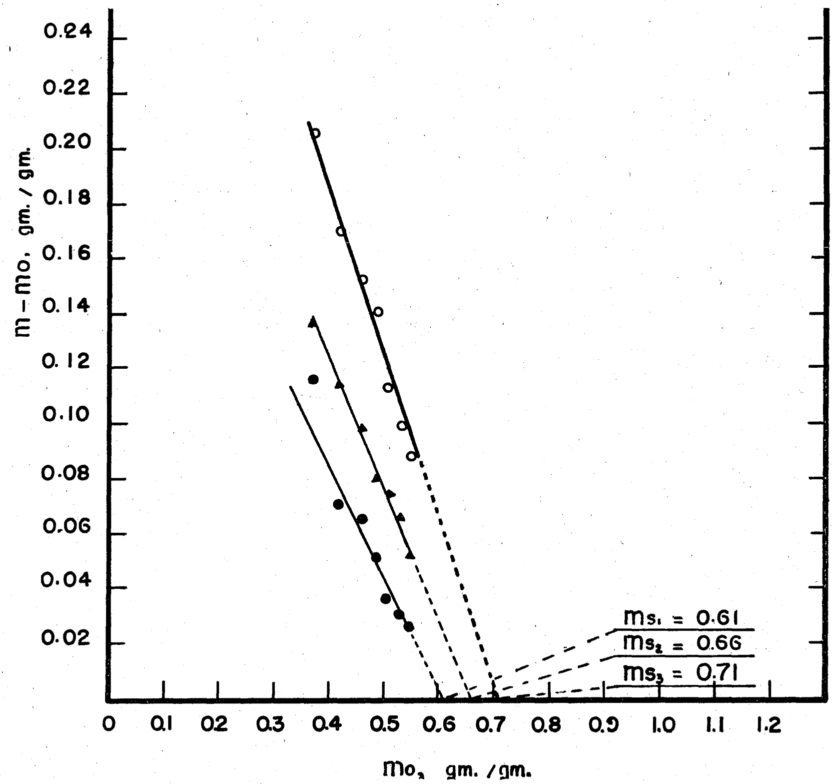

Fig. 4. Extrapolation of $m-m_o$ as a function of m_o at different temperature to obtain the effective surface moisture content m_s for white kafir (grain sorghum). Time, 15 min. Open circles, 160° F.; solid circles, 100°F.; triangles, 130°F.

For popcorn, m_s was 0.515 g. per g. at 100°F., and 0.545 g. per g. at 160°F. The numerical values were nearly constant, hence the average value of 0.530 was used for the general correlation.

For white kafir, the values were 0.61 g. per g. at 100°F., 0.66 g. per g. at 130°F., and 0.71 g. per g. at 160°F. The values at other temperatures were therefore estimated by linear interpolation and extrapolation. The values of m_s for other samples are listed in Table II.

Surface Area-to-Volume Ratio S/V. The surface area-to-volume ratio of the samples can be evaluated from their sphericity and volume as (5):

$$\left(\frac{S}{V}\right) = \frac{4.83}{\psi V^{1/3}} \tag{8}$$

The surface area-to-volume ratio of each sample can be evaluated from the known value of ψ and V according to equation 8.

The initial surface-volume ratio of each sample was used to analyze and correlate the experimental results because the diffusion model employed was derived by assuming that the ratio of S/V is constant. The sphericity, ψ, is related to the bed porosity, which is dimensionless, of the grains (4). The experiments were, therefore, carried out to determine the initial porosity for each sample. These were 0.3647 for popcorn, 0.4178 for K-1859 hybrid corn, 0.4365 for white kafir sorghum, 0.5205 for sweet corn, and 0.4210 for Atlas sorgo sorghum.

TABLE II
EFFECTIVE SURFACE MOISTURE CONTENTS

Material	Temperature	m_s
	°F.	g./g.
K-4 hybrid popcorn	100	0.515
	160	0.545
K1859 hybrid corn (dent corn)	84	0.45
	150	0.45
Gold Rash sweet corn	84	0.73
	120	0.73
	150	0.73
White kafir sorghum	100	0.61
	130	0.66
	160	0.71
Atlas sorgo sorghum	100	0.500
	130	0.525
	160	0.545

General Correlation. If the diffusion model used in this work is valid there should be a linear relationship between the experimentally obtained dimensionless weight gain, $(1-\overline{M})$, and the variable X with a slope of $2/\sqrt{\pi}$. To test this, the data were correlated in such a manner. The results are plotted in Figs. 5 and 6 for popcorn and white kafir sorghum respectively. The experimental data obtained at all the temperature levels lie in straight lines as predicted. The deviation between the experimental results and the correlation lines might be caused by the following factors: a) the surface area-volume ratio of the solid during steeping is not a constant; b) the applicability of the constant value of m_s; c) the inherent inaccuracy associated with the experiment.

Nevertheless, equation 5 may be considered a satisfactory correlation model for the diffusion of water into kernels of corn and sorghum.

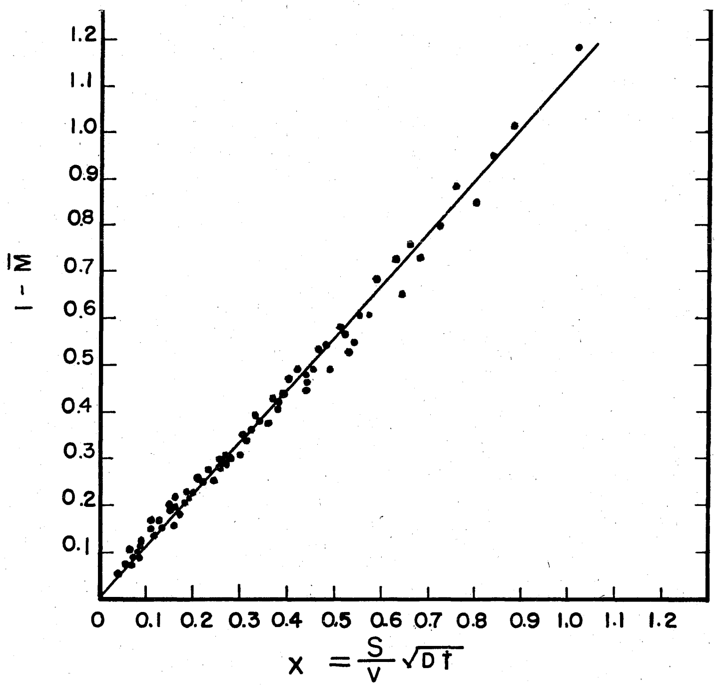

Fig. 5. Dimensionless correlation for the weight gain data of the K-4 hybrid popcorn according to the first-order approximation to diffusion equation.

Evaluation of the Diffusion Coefficient D. Equation 5 was rearranged as:

$$D = \left[\frac{K}{\frac{2}{\sqrt{\pi}} \left(\frac{S}{V}\right) (m_s - m_o)} \right]^2 \qquad (9)$$

The diffusion coefficient, D, then, could be evaluated from equation 9 with the known quantities of (S/V), $(m_s - m_o)$, and K which is the slope of the linear correlation of $(\overline{m} - m_o)$ vs. $\sqrt{t}$.

The values of D for all the five different cereal grains are plotted against the reciprocal of the absolute temperature on a semilogarithmic scale in Figs. 7 and 8. The figures show that the relation between the diffusion coefficients and the absolute temperature follows the Arrhenius equation:

$$D = D_o \exp \left[- \frac{E}{RT} \right] \qquad (10)$$

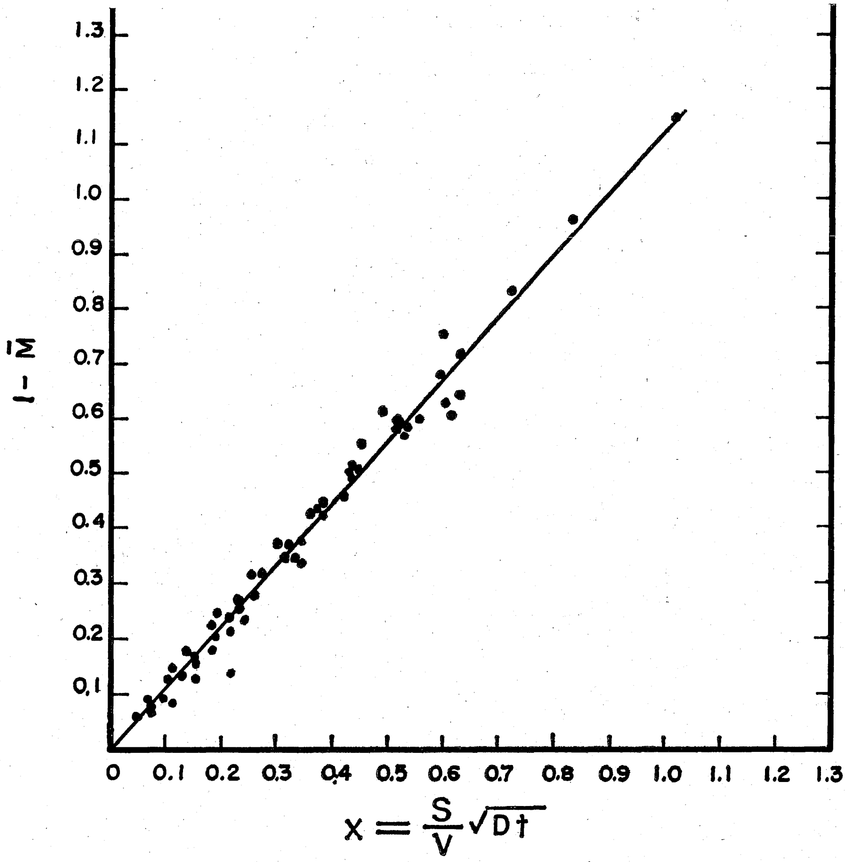

Fig. 6. Dimensionless correlation for the weight gain data of white kafir (grain sorghum), according to the first-order approximation to diffusion equation.

where E is the energy of activation, R is the universal gas constant, and T is the absolute temperature. The constants, D_0, and the slopes (E/R) of the linear regression lines of the Arrhenius relation were estimated by the method of least squares, and the energy of activation E was evaluated from them. The results are summarized below:

Popcorn	E = 6.853 kcal./mol.	$D_o = 1.535 \times 10^{-2}$ cm.^{2}/sec.
K-1859 hybrid corn	E = 7.578 kcal./mol.	$D_o = 1.077 \times 10^{-1}$ cm.^{2}/sec.
Gold Rash sweet corn	E = 8.167 kcal./mol.	$D_o = 8.535 \times 10^{-2}$ cm.^{2}/sec.
White kafir sorghum	E = 8.339 kcal./mol.	$D_o = 4.47 \times 10^{-2}$ cm.^{2}/sec.
Atlas sorgo sorghum	E = 8.424 kcal./mol.	$D_o = 6.196 \times 10^{-2}$ cm.^{2}/sec.

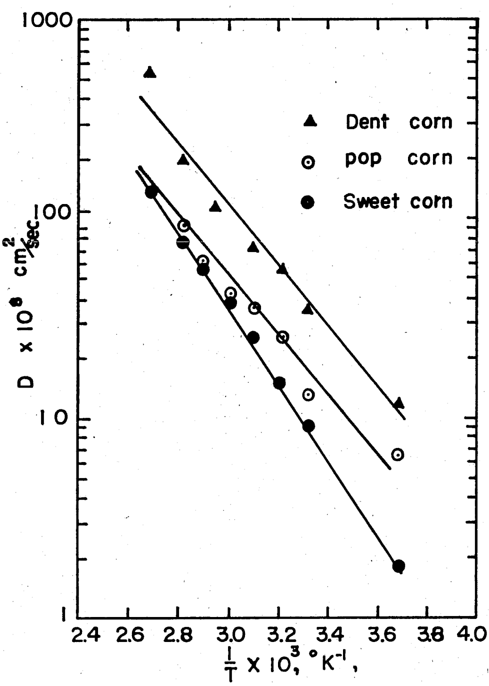

Fig. 7. The diffusion coefficients as a function of the reciprocal of absolute temperature for corn.

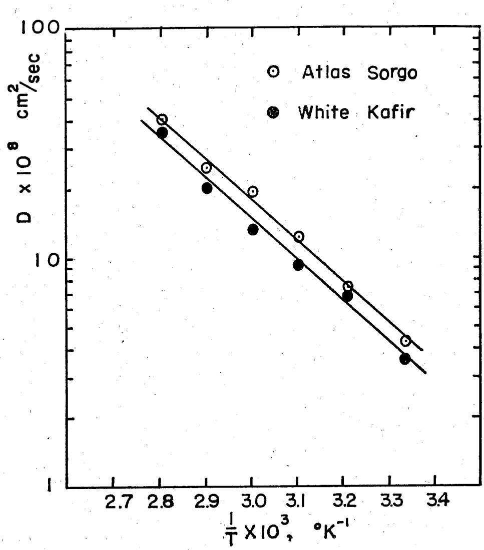

Fig. 8. The diffusion coefficients as a function of the reciprocal of absolute temperature for sorghum.

Conclusion

The results obtained in this work show that the experimentally measured steeping data of all the five samples obey the approximate integrated diffusion equation based on Fick's law. The equation holds for all temperatures from 0° to 100°C. The validity of such an equation was hitherto tested only for wheat.

The diffusion coefficients, D, were evaluated from the experimental data and plotted as a function of the reciprocal of the absolute temperature on a semilogarithmic scale. The results show that the relation between D and 1/T follows the Arrhenius equation.

It was also shown that the diffusivities of water in the grain of corn and sorghum are of the same order of magnitude as those in wheat.

The diffusion equation tested and the empirical diffusivities evaluated in the present work would be of considerable practical value because of the extensive use of the steeping operation in processing corn and sorghum.

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